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NONSTEADY SHOCK WAVES IN GAS – LIQUID MIXTURES OF BUBBLE STRUCTURE

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In recent years many theoretical and experimental reports have been published on the investigation of shock waves in liquids containing gas bubbles [1-11]. The reports on the experimental level are devoted to the investigation of the structures of compression waves in mixtures with rather large bubbles (~1 mm) [4-6, 9]. Because of the considerable lengths of the relaxation zones for waves in such mixtures (~1 m), comparable with the lengths of the shock tubes, the waves observed in [4-6, 9] were nonsteady, as a rule. This was first noted in [8], where the necessity of enlisting the nonsteady theory in the analysis of experimental data was pointed out (up to then only steady wave configurations were studied in the theoretical reports [1, 3-5, 8, 9]). The propagation of a weak nonsteady wave was first studied in [7] on the basis of the Burgers–Korteweg–de Vries model equation. The report [10] is devoted to a description of the general approach to the investigation of nonsteady waves in bubble media. In the present report principal attention is paid to questions of the concrete definition of the model of the dynamic behavior of the medium and to a discussion of recent results.

§1. To describe the nonsteady motions of mixtures of liquids and gas bubbles we use the methods of the mechanics of a continuous medium, assuming that the characteristic linear scales of the flow are much larger than the sizes of the bubbles and the distances between them. We construct the model of the dynamic behavior of the mixture with the following simplifying assumptions:

1. The viscosities and thermal conductivities of the phases are important only in processes of interaction between the phases.
2. The bubbles are spherical and monodisperse.
3. Breaking up, collisions, and coagulation of bubbles are absent.
4. The velocities of the macroscopic motions of the phases coincide.
5. The density and temperature of the liquid are constant.

Let us discuss assumptions 4 and 5, which are of fundamental interest from the point of view of simplicity of the solution of concrete problems, in more detail. In sufficiently weak waves the difference in the velocities of the phases is small and viscous dissipation in the relative translational motion of the liquid and bubbles is barely noticeable against the background of the dominant thermal dissipation [8]. In stronger waves, when the noncoincidence of the velocities of the phases is significant, the bubbles break up, as a rule [12]. This leads to a sharp decrease in the slipping of the phases and to a corresponding decrease in the dissipation due to the relative motion. With allowance for the effect of the breaking up of bubbles, one can also study rather strong waves within the framework of the one-velocity approach.

The assumption of constancy of the liquid temperature is fully justified from the physical point of view, since the heat capacity of the liquid (per unit volume of the mixture) considerably exceeds the heat capacity of the gas. The assumption of constancy of the liquid density is applicable if the volume content of bubbles in the mixture is high enough and the compressibility of the mixture is practically determined by the deformation of its gaseous component.

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The assumptions that the density and temperature of the liquid are constant allow one to considerably simplify the problem of the investigation of nonsteady flows of the mixture. This pertains especially to the assumption that the liquid phase is incompressible, the absence of which complicates the modeling of processes of wave propagation, since it then becomes necessary to make a detailed calculation of the radial motions of individual bubbles, of the fields of all the parameters near them, and of the processes of damping of the shock discontinuities initiated in the carrier phase.

With the assumptions made, we can write the differential laws of conservation of mass, momentum, and energy of the mixture in one-dimensional nonsteady motion in the presence of the external force of gravity. The equations of conservation of mass and momentum of the mixture have the form

$$\begin{gathered}\frac{d\rho_1}{dt}+\rho_1\frac{\partial v}{\partial x}=0,\quad \frac{d\rho_2}{dt}+\rho_2\frac{\partial v}{\partial x}=0,\\ \rho\frac{dv}{dt}+\frac{\partial p}{\partial x}=\rho g,\\ \rho_1=\alpha_1\rho_1^0,\quad \rho_2=\alpha_2\rho_2^0,\quad \rho_1^0=\mathrm{const},\quad \alpha_1+\alpha_2=1,\\ \rho=\rho_1+\rho_2,\quad p=\alpha_1 p_1+\alpha_2(p_2-4\sigma/\delta),\end{gathered}\tag{1.1}$$

where ρ, p, and v are the mean density, mean reduced pressure, and mean velocity of the mixture; ρ_i^0, ρ_i, p_i, and α_i are the true and mean density, the pressure, and the volume content of the carrier phase; g is the acceleration of gravity; the subscripts 1 and 2 pertain to the parameters of the liquid and the gas, respectively.

The equations of the inflow of heat to the phases can be written in the form

$$\rho_2\frac{du_2}{dt}=\frac{\alpha_2 p_2}{\rho_2^0}\frac{d\rho_2^0}{dt}+nq,\quad T_1=\mathrm{const},\tag{1.2}$$

where u_2 is the internal energy of the gas; q is the intensity of heat exchange between phases per bubble; n is the number of bubbles per unit volume of the mixture; T is the absolute temperature.

In accordance with assumption 5 we assume that the liquid is incompressible, and as the equations of state of the second phase we take the equations of state of a calorifically ideal gas

$$\rho_1^0=\mathrm{const},\quad p_2=(\gamma-1)c_{V2}\rho_2^0 T_2,\quad u_2=c_{V2}T_2,\tag{1.3}$$

where γ is the adiabatic index; c_{V2} is the specific heat of the gas at constant volume.

By virtue of the adopted assumptions 2 and 3, the following equations are valid:

$$\rho_2^0\delta^3=\mathrm{const},\quad \alpha_2=\pi\delta^3 n/6.\tag{1.4}$$

The intensity of the heat exchange between an individual bubble and the liquid will be taken as proportional to the difference in temperatures of the phases:

$$q=\pi\delta^2\beta(T_1-T_2)=\pi\delta\lambda_2\,\mathrm{Nu}\,(T_1-T_2)(\mathrm{Nu}=\beta\delta/\lambda_2),\tag{1.5}$$

where δ is the bubble diameter; β and Nu are the coefficient of heat exchange between phases and the Nusselt number; λ_2 is the coefficient of thermal conductivity of the gas.

The system of equations (1.1)-(1.5) is closed if the conditions of codeformation of the components of the mixture are assigned. The Rayleigh–Lamb equations [13, 14] are usually used as such conditions for a liquid containing bubbles. It is known, however, that these equations were obtained in application to the oscillations of a single bubble located in an unbounded liquid. If the pulsating bubble is not alone (is in the vicinity of an ensemble of other bubbles), then one can not get by without allowance for their influence on the dynamics of its radial motion. The appropriate corrections to the Rayleigh equation for the "gas content" can be obtained within the framework of a cell model of the medium, analogous to the corresponding model in the kinetic theory of dense gases.

We will be confined to the consideration of cells of spherical shape with bubbles located at their geometrical centers. We will assume that disturbances whose sources are located outside the cells do not affect the flow of liquid within them (and vice versa). We have

$$(\delta/2R)^3=\alpha_2,\quad \varphi=-w\delta^2/4r,\quad v=w\delta^2/4r^2,$$

where R is the radius of an equivalent cell; φ and v are the potential and velocity of the radial motion of the liquid, described by the Cauchy–Lagrange integral

$$\frac{\partial\varphi}{\partial t}+\frac{v^2}{2}+\frac{p}{\rho_1^0}=-\frac{\delta}{2}\frac{\partial w}{\partial t}-\frac{3}{2}w^2+\frac{(p_2-4\sigma/\delta)}{\rho_1^0}. \tag{1.6}$$

We introduce the mean macroscopic pressure in the liquid:

$$p_1=3\left(\int_{\delta/2}^{R}pr^2dr\right)\Big/(R^3-(\delta/2)^3).$$

Integrating (1.6) over the volume of a cell and allowing for the viscosity of the liquid, we obtain the following refined condition of codeformation of the phases (the equation for the pulsating motion of a bubble in the mixture):

$$\begin{gathered}\delta(1-\varphi_1)\frac{\partial w}{\partial t}+3(1-\varphi_2)w^2+16\frac{\nu_1}{\delta}w=\frac{2(p_2-p_1-4\sigma/\delta)}{\rho_1^0},\quad \frac{\partial\delta}{\partial t}=2w,\\ \varphi_1=3\left(\alpha_2^{1/3}-\alpha_2\right)/2\alpha_1,\quad \varphi_2=\left(\alpha_2^{1/3}(2+\alpha_2)-3\alpha_2\right)/\alpha_1,\end{gathered} \tag{1.7}$$

where φ_1 and φ_2 are unknown correction factors allowing for the influence of the finiteness of the volume content of bubbles on the character of their pulsating motion. They are proportional to $\alpha_2^{1/3}$ in order of magnitude, and they can prove significant at high enough volume contents of gas ($\alpha_2 \gtrsim$ 1-3%). As $\alpha_2 \to 0$ we have φ_1, $\varphi_2 \to 0$ and Eq. (1.7) changes into the ordinary Rayleigh–Lamb equation.

§2. For the numerical modeling on a computer of the nonsteady wave processes in liquids containing gas bubbles, we change to the following dimensionless variables and parameters:

$$\begin{gathered}P_i=p_i/p_0,\quad V=v/a_*,\quad \Phi=\rho/\rho_{10}^0,\quad \Phi_i^0=\rho_i^0/\rho_{10}^0,\quad \Theta_i=T_i/T_0,\\ \tau=ta_*,\quad C_2=c_{V2}T_0/a_*^2,\quad \bar\nu_1=16\nu_1/a_*,\quad \bar\sigma=4\sigma/p_0,\\ \beta_*=6\,\mathrm{Nu}\,\lambda_2/\rho_{20}^0c_{V2}\delta_0^3a_*,\\ W=w/a_*\ \left(p_0=p_{10}(0),\ a_*^2=p_0/\rho_{10}^0,\ T_0=T_{10}=T_{20}\right).\end{gathered} \tag{2.1}$$

Following the concept of [10], we transform the initial system of differential equations (1.1), (1.2), (1.7) to a form convenient for integration. Instead of the Eulerian coordinates (x, t) we use the Lagrangian coordinates (r, τ), since they are more convenient for the solution of problems of this class within the framework of the one-velocity model constructed. In the variables (2.1) the closed transformation of the system of equations for the description of nonsteady waves in a liquid containing bubbles in the Lagrangian coordinates has the form

$$\begin{gathered}\frac{\partial^2P}{\partial r^2}-\frac{1}{\Phi_0}\frac{d\Phi_0}{dr}\frac{\partial P}{\partial r}=-6\Phi_0^2\frac{\alpha_2}{\Phi\delta^2(1-\varphi_1)}[(1-4\varphi_1+3\varphi_2)W^2-\bar\nu_1W/\delta+2(P_2-P-\bar\sigma/\delta)/\alpha_1],\ \frac{dV}{dr}=6\Phi_0\frac{\alpha_2W}{\Phi\delta},\\ \frac{\partial\Phi}{\partial\tau}=-6\frac{\alpha_2W\Phi}{\delta},\quad \frac{\partial\Theta_2}{\partial\tau}=6(1-\gamma)\frac{W\Theta_2}{\delta}+\beta_*\delta(1-\Theta_2),\\ \partial W/\partial\tau=\left[2\left(P_2-P-\bar\sigma/\delta\right)/\alpha_1-3(1-\varphi_2)W^2-\bar\nu_1W/\delta\right]/\delta(1-\varphi_1),\\ \partial\delta/\partial\tau=2W,\\ P_2=C_2(\gamma-1)\Phi_2^0\Theta_2,\quad \alpha_2=(1-\Phi)/(1-\Phi_2^0),\quad \Phi_2^0=\Phi_{20}^0\delta_0^3/\delta^3,\\ \alpha_1=1-\alpha_2,\\ \varphi_1=3\left(\alpha_2^{1/3}-\alpha_2\right)/2\alpha_1,\quad \varphi_2=\left[\alpha_2^{1/3}(2+\alpha_2)-3\alpha_2\right]/\alpha_1.\end{gathered} \tag{2.2}$$

It consists of six differential equations, each of which contains only one derivative with respect to one of the coordinates (r or τ). The first two equations of the system serve for the determination of the reduced pressure and velocity of the mixture at an arbitrary time from the known fields of the remaining parameters; the other equations describe the laws of variation of the parameters of the Lagrangian particles of the medium with time.

For the numerical integration of the system (2.2) we divide a volume of the medium defined by the points $r_1, r_2, \ldots, r_n$ into n material particles: the values of all the unknown functions will be determined at the points $r = r_i$ ($i = 1, 2, \ldots, n$). Then the last four differential equations in partial derivatives of the variables Φ, Θ_2, W, and δ with respect to time change into 4n ordinary differential equations, for the numerical integration of which it is convenient to use the modified Euler–Cauchy method [15]. To determine the values of the pressure P at the points $r = r_i$ at each fixed time one must solve the boundary problem for the first differential equation of (2.2) with the following conditions at the boundaries of an isolated volume of the mixture ($r = 0$, $r = l$):

$$\begin{gathered}r=0\ \ P(0,\tau)=P_0(\tau)\ \text{ or }\ \partial P/\partial r(0,\tau)=\chi_0(\tau),\\ r=l\ \ P(l,\tau)=P_l(\tau)\ \text{ or }\ \partial P/\partial r(l,\tau)=\chi_l(\tau).\end{gathered}$$

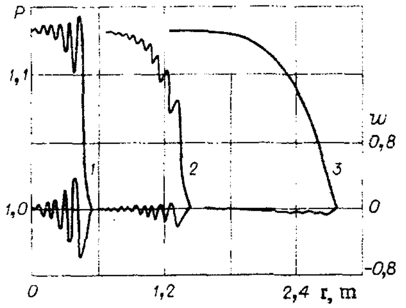

Fig. 1

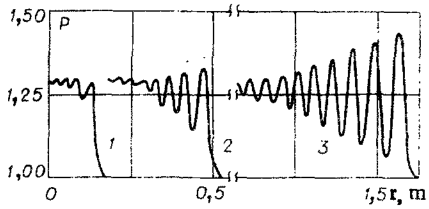

Fig. 2

It is expedient to use the trial-run method [16] to solve this problem. We note that the velocity of the medium enters only into the second equation of (2.2), so that is not obligatory to calculate it at each step of the integration in time.

§3. The nonsteady wave processes in a 50% solution of glycerine in water containing gas bubbles (air or helium) were modeled numerically. We studied the basic laws of the evolution of the structures of nonsteady shock waves, using the following values of the thermodynamic parameters of the phases ($p_0 \cong 1$ bar, $T_0 = 300°K$)

$$\text{liquid:}\quad \rho_{10}^0 = 1126\ \text{kg/m}^3,\quad \nu_1 = 0.75\cdot 10^{-5}\ \text{m}^2/\text{sec};$$
$$\text{air:}\quad \gamma = 1.4,\quad \rho_{20}^0 = 1.21\ \text{kg/m}^3,\quad \lambda_2 = 0.025\ \text{kgm/(sec}^3\cdot\text{deg)},\quad c_{V2} = 716\ \text{m}^2/(\text{sec}^2\cdot\text{deg}),$$
$$\text{helium:}\quad \gamma = 1.66,\quad \rho_{20}^0 = 0.16\ \text{kg/m}^3,\quad \lambda_2 = 0.151\ \text{kgm/(sec}^3\cdot\text{deg)},\quad c_{V2} = 3128\ \text{m}^2/(\text{sec}^2\cdot\text{deg}).$$

It was established that the properties of the evolution of a shock wave in a bubble medium depend strongly on the effects of thermal dissipation. In performing concrete calculations one must successively allow for the influence of the parameters of the mixture, the thermophysical properties of the gas and the bubbles, and the intensity of the waves on the parameter Nu of internal heat exchange. This allowance can be made within the framework of the recommendations of [8] for a steady-state analysis. In accordance with them the parameter Nu and the coefficient of heat exchange β between the phases under the conditions of pulsating motion of the bubbles are determined by the relations

$$\mathrm{Nu} = \delta/(h_2 t_*)^{1/2},\quad \beta = \lambda_2/(h_2 t_*)^{1/2},\quad h_2 = \lambda_2/c_{V2}\rho_2^0,$$
$$\pi\delta_0/a_*(3(\gamma p_e/p_0 - 1))^{1/2} \leqslant t_* \leqslant \pi\delta_0/a_*(3(p_e/p_0 - 1))^{1/2}, \tag{3.2}$$

where t_* is the characteristic time of pulsations of the bubbles. These very equations were used to determine Nu in the numerical experiments conducted. The calculations showed that in water–air mixtures with a bubble size on the order of 1 mm ($p_0 \cong 1$ bar) the structure of sufficiently weak waves ($p_e/p_0 \lesssim 1.4$) evolves from an oscillating to a monotonic structure. The structure of stronger waves† ($p_e/p_0 \gtrsim 1.4$) approaches an extreme oscillating configuration in the course of evolution. The evolution of waves in mixtures of bubbles with liquids of moderate viscosity [4, 5, 9] is due to the effects of heat exchange between phases and of the transfer of the kinetic energy of radial motion into neighboring volumes of the mixture due to the pressure disturbance (but not to the effects of viscosity in the relative motion of the phases, as asserted in [9]). As an example, in Fig. 1 we present profiles of the pressure and radial velocity of bubbles at different times in a shock wave of intensity $p_e/p_0 = 1.13$ propagating through a mixture with the following parameters: $p_0 = 1.045$ bar, $\alpha_{20} = 1.7\%$, $\delta_0 = 2.5$ mm; curve 1) t = 5; 2) 15; 3) 30 msec. It is seen that the steady configuration of this wave is monotonic and is formed at a distance of about 3 m in a time on the order of 30 msec.

Calculations were made in order to study the influence of the initial pressure p_0 of the mixture on the process of evolution of the structures of shock waves. We examined the motion of shock waves of the same dimensionless intensity p_e/p_0 in the same mixtures when the pressure p_0 was varied in the range of 0.1-10 bar. It was established that an increase in p_0 leads not only to an increase in the wave velocity (the wave velocity varies in proportion to $p_0^{1/2}$ [8]), but also to an increase in the amplitudes and lengths of the oscillations in the front. The latter is connected with the fact that the intensity of thermal dissipation in the process of heat exchange between phases decreases with an increase in pressure, by virtue of the fact that the characteristic

†The propagation of very strong shock waves ($p_e/p_0 \approx 10^2$-10^3) can be studied within the framework of models of media with second viscosity [17].

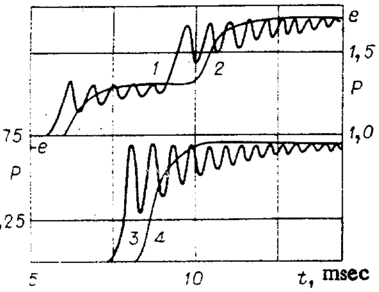

Fig. 3

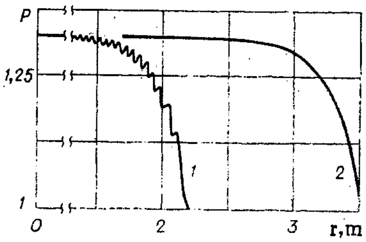

Fig. 4

dimensionless parameter β_* of heat exchange [see (2.1)] is proportional to $p_0^{-3/4}$ [$\rho_{20}^0 \sim p_0$, $a_* \sim p_0^{1/2}$, $t_* \sim p_0^{-1/2}$, $Nu \sim p_0^{3/4}$ (see (3.2)), so that $\beta_* \sim p_0^{-3/4}$].

The results of the calculations are illustrated in Fig. 2, where we present the pressure profiles in nonsteady shock waves of intensity $p_e/p_0 = 1.3$, formed 6 msec after their initiation by a piston moving with a constant velocity in a mixture with $\alpha_{20} = 2.5\%$ and $\delta_0 = 3$ mm. Curve 1) $p_0 = 0.1$; 2) 1; 3) 10 bar.

We analyzed the properties of the reflection of nonsteady shock waves from rigid walls. As an example, we solved the problem of the motion of a wave initiated at the boundary of an isolated volume of the mixture with a pressure $p_0 = 1$ bar through the instantaneous increase of the pressure at the boundary to $p_e = 1.3$ bar. We calculated the process of reflection of the wave from a wall located at a distance of 1 m from the point of initiation. The results of the calculations for mixtures with $\alpha_{20} = 1\%$ and $\delta_0 = 3$ mm and $\delta_0 = 1$ mm are presented in Fig. 3 in the form of pressure oscillograms "recorded" at a distance of 0.25 m from the wall (curves 1 and 2) and at the wall itself (curves 3 and 4). Curves 1 and 3 are $\delta_0 = 3$ mm and 2 and 4 are 1 mm; the equilibrium pressure behind the reflected wave is marked by the letter e on the ordinate. It is seen that the lengths of the relaxation zones decrease with a decrease in bubble diameter. At a bubble size of 1 mm the waves have almost monotonic structures.

It is known that the reflection of low-intensity shock waves (when the compressibility of the liquid component of the mixture can be neglected) takes place in accordance with the law

$$p_{2e}/p_0 = (p_e/p_0)^2,$$

where p_e and p_{2e} are the equilibrium pressures behind the incident and reflected waves, respectively. In experiments on shock tubes, however, the above-mentioned p_{2e} cannot always be recorded because of the specifics of the experiment itself. If the length of the relaxation zone of the incident wave is large, then the rarefaction wave from the high-pressure chamber of the shock tube can reach the opposite wall before the time of establishment of the equilibrium pressure. Then the maximum recorded pressure p_* at the wall will be lower than its expected equilibrium value.

Thus, the p_* recorded in the experiments can depend on a whole series of factors: the initial size of the bubbles, the thermophysical properties of the gaseous phase, the intensity of the wave, the presence or absence of effects of breaking up of bubbles, etc. Other conditions being equal, the length of the high-pressure chamber of the shock tube can also affect the value of p_*. With a sufficiently intense incident wave the bubbles break up, and the extent of the relaxation zone (diffuseness of the profile) of the wave is sharply reduced. In this case the maximum equilibrium pressures behind the reflected waves are able to be established, as a rule, before the arrival of the rarefaction waves. In the case of the absence of breaking up of the bubbles in the wave the situation is greatly altered, since the diffuseness of the incident wave front grows strongly.

We made calculations in application to the experimental data of [4, 5, 9]. We studied the evolution of waves of intensity $p_e/p_0 > 1 + 2\alpha_{20}$. The analysis of [9], carried out without allowance for the effects of thermal dissipation in the radial motion of the bubbles, showed that steady waves of such an intensity must have an oscillating structure. Waves with an intensity $p_e/p_0 > 1 + 2\alpha_{20}$ observed in the experiments actually were oscillating waves. In this connection the hypothesis was advanced in [8] that these were oscillating waves only by virtue of their nonsteadiness (it was impossible to observe steady wave configurations under the experimental conditions of [4] because of the insufficient length of the shock tube). The results of the calculations of the process of evolution of nonsteady waves confirmed this hypothesis. In Fig. 4 we give an example of a calculation of the evolution of the pressure profile in a wave of $p_e/p_0 = 1.32$, carried out in application to the experimental conditions of [4]: $p_0 = 0.902$ bar, $\alpha_{20} = 2.5\%$, and $\delta_0 = 0.28$ mm. It is seen that a monotonic wave structure is realized at a distance of more than 3 m (in the experiments of [4] the detectors were located at

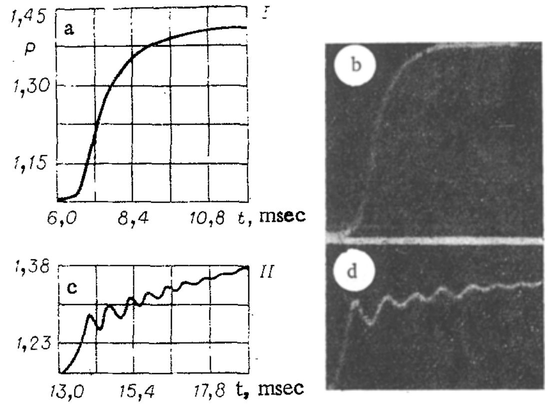

Fig. 5

distances of less than 1.6 m from the point of initiation of the waves and therefore only nonsteady wave configurations were recorded). An analysis of the aggregate of experimental data of [4, 6, 9] shows that the majority of the results obtained pertain to nonsteady waves, so that their correct analysis can be carried out only with the enlistment of the nonsteady theory.

We studied the influence of the thermophysical properties of the gas bubbles on the process of evolution of the structures of the shock waves. It was established that the thermophysical properties of the gas can strongly affect the characteristic time of thermal relaxation and, in this connection, the development of the evolutionary processes as a whole. Let us trace this influence on the example of two gases, air and helium, having considerably different thermophysical properties [see (3.1)]. In accordance with (3.1) and (3.2) the dimensionless coefficients of heat exchange β_{*a} and β_{*h} of air and helium with the liquid under the conditions of a pulsating motion of the bubbles are connected by the relation

$$\beta_{*h} \cong 3.5\beta_{*a}.$$

For just this reason the intensity of thermal dissipation must be greater in mixtures containing helium bubbles than in a mixture containing air bubbles: accordingly, other conditions being equal, the rate of formation of monotonic wave configurations is also greater. The results of the calculations and their comparison with the experimental data confirm this conclusion. As an example, in Fig. 5 we present (at the same scale) calculated and experimental† oscillograms of the pressure in shock waves I (a, b) and II (c, d): I (recording at a depth of 0.82 m from the surface of the mixture; the gas is helium): $p_e/p_* = 1.3$, $p_* = 1.09$ bar, $\alpha_{2*} = 1\%$, $\delta_* = 1.5$ mm; II (recording at a depth of 1.59 m from the surface of the mixture; the gas is air): $p_e/p_* = 1.18$, $p_* = 1.09$ bar, $\alpha_{2*} = 1\%$, $\delta_* = 1.9$ mm. Here the values of the parameters at a depth of 0.82 m are marked by an asterisk. It is seen that the shock wave in the mixture containing helium bubbles already has a monotonic structure at a depth of 0.82 m, whereas a weaker wave in a practically analogous mixture containing air bubbles still has a clearly expressed oscillating structure at the considerably greater depth of 1.59 m. The calculated oscillograms are in satisfactory agreement with the experimental ones.

The calculations made and their comparison with experimental data show that the constructed model of the dynamic behavior of a mixture can be used successfully for an adequate description of nonsteady wave processes in liquids containing gas bubbles.

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ACTION OF A PRESSURE PULSE ON A CAVITY IN A VISCOUS LIQUID

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The case of the collapse of a cavity under the action of a constant external pressure p_0 was analyzed in [1]. There is a class of problems, however, in which the external action consists of brief pressure pulses. Such a situation occurs, for example, in the impact loading of porous solids.

Suppose that there is an empty spherical cavity of radius r_0 in a viscous incompressible liquid with a density ρ. The pressure $p_\infty(t, \tau)$ at infinity (far from the cavity) is an arbitrary function of time at $0 \le t \le \tau$ and is reduced to zero at $t > \tau$.

The motion is spherically symmetric and the Navier–Stokes equations describing it have the form

$$\frac{\partial u}{\partial r} + 2\frac{u}{r} = 0, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \tag{1}$$

where $u(r, t)$ is the velocity; $p(r, t)$ is the pressure.

At the surface of the cavity a normal stress σ_{rr} is absent (the cavity is empty), and since $\sigma_{rr} = -p + 2\eta du/dr$, we have $p_1 = 2\eta(\partial u/\partial r)_1$. Here and later the values of quantities at the boundary are marked by the index 1; η is the coefficient of dynamic viscosity.

The second boundary condition will be

$$p = p_\infty(t, \tau) \quad \text{at} \quad r = \infty.$$

From the first equation of (1) we obtain $u(r, t) = u_1 r_1^2/r^2$.

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